Resource allocation strategies for multicarrier radio systems

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Summary

- Introduction
  - Convex optimization
- Single-cell resource allocation
  - Rate adaptive
  - Margin adaptive
  - Optimal allocation
  - MIMO
- Multi-cell resource allocation
  - Distributed
  - Central coordination
  - Static

Introduction

OFDMA resource allocation schemes

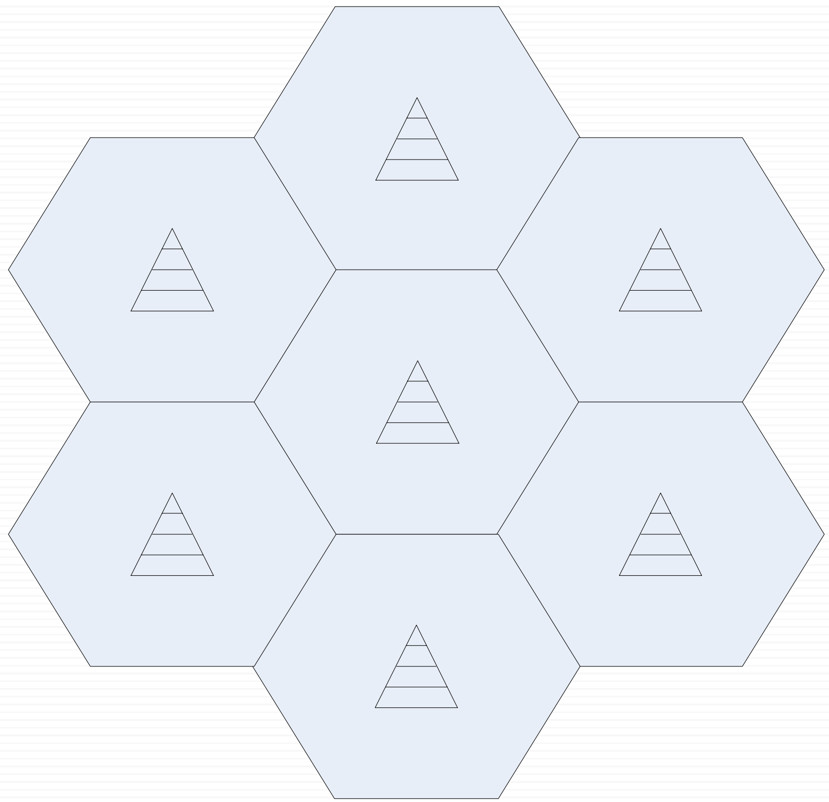
- Orthogonal Frequency Division Multiplexing (OFDM) is one of the most adopted modulation techniques by current air interface standards (e.g. 802.16, 3GPP Long Term Evolution)
  - OFDM is robust to the multi-path wireless propagation channel
  - In OFDMA systems it is possible to exploit channel frequency diversity by dynamically assigning the radio resources to the users.

System model

- OFDMA *synchronized* system
  - N_c cells
  - N subcarriers
  - K users per cell
- Received signal for user k in cell i on channel n

$$X_{k,n}^{(i)} = H_{k,n,i}^{(i)} S_n^{(i)} + d_{k,n}^{(i)}$$

$$d_{k,n}^{(i)} \in \mathcal{N}(0, \sigma^2 + I_{k,n}^{(i)})$$



OFDMA signal model

- The received signal-to-interference-plus- noise ratio (SINR) $\gamma_{k,n}{}^{(i)}$ is ($G_{k,n,i}{}^{(i)}=|H_{k,n,i}{}^{(i)}|^2$)

$$\gamma_{k,n}^{(i)} = \frac{P_n^{(i)} G_{k,n,i}^{(i)}}{\sigma^2 + I_{k,n}^{(i)}}$$

- The power of the MAI $I_{k,n}{}^{(i)}$, affecting user k in cell i on the n channel, is given by

$$I_{k,n}^{(i)} = \sum_{j=1, j\neq i}^{N_c} P_n^{(j)} G_{k,n,i}^{(j)}$$

- Accordingly, the capacity of user k in cell i on channel n

$$C_{k,n}^{(i)} = \log_2\left(1 + \gamma_{k,n}^{(i)}\right)$$

Definition of convexity

- $f: \mathcal{D} \to \mathcal{R}$ is *convex* if $\mathcal{D}$ is a convex set and

$$f(\theta x + (1-\theta)y) \le \theta f(x) + (1-\theta) f(y)$$

for any $x, y \in \mathcal{D}$ and $0 \le \theta \le 1$

- **Example**: function $f(x) = \max_i x_i$

$$\begin{aligned} f(\theta x + (1-\theta)y) &= \max_i (\theta x_i + (1-\theta) y_i) \\ &\le \theta \max_i x_i + (1-\theta) \max_i y_i \\ &= \theta f(x) + (1-\theta) f(y) \end{aligned}$$

Convex optimization

- **Standard form** *convex* optimization problem

$$\min f_0(x)$$
$$s.t.$$
$$f_i(x) \le 0 \quad i = 1, \ldots, m$$
$$Ax = b$$

  - $f_0, f_1, \cdots, f_m$ are convex;
  - equality constraints are affine

Convex optimization

- **Standard form** problem (not necessarily convex)

$$\min f_0(x)$$
$$s.t.$$
$$f_i(x) \le 0 \quad i = 1,\dots,m$$
$$h_i(x) = 0 \quad i = 1,\dots,p$$

Variable $x \in \Re^n$, domain $\mathcal{D}$, optimal value $p^* = f_0(x^*)$

- **Lagrangian**

$$L(x,\lambda,\nu) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x)$$

  - Weighted sum of objective and constraint functions

Convex Optimization

□ Lagrangian dual function g

$$g(\lambda,\nu) = \inf_{x\in\mathcal{D}} L(x,\lambda,\nu) = \inf_{x\in\mathcal{D}}\left(f_0(x) + \sum_{i=1}^{m}\lambda_i f_i(x) + \sum_{i=1}^{p}\nu_i h_i(x)\right)$$

- g is *affine* in λ and ν and therefore concave AND convex
- **Lower bound property**: if $\lambda \geq 0$, then $g(\lambda,\nu) \leq p^*$

  Proof: if x_0 is feasible and $\lambda \geq 0$, then

  $$f_0(x_0) \geq L(x_0,\lambda,\nu) \geq \inf_{x\in\mathcal{D}} L(x,\lambda,\nu) = g(\lambda,\nu)$$

  minimizing over all feasible x_0 gives $p^* \geq g(\lambda,\nu)$

The dual problem

- **Lagrange dual problem (LDP)**

$$\max g(\lambda, \nu)$$
$$s.t.$$
$$\lambda \geq 0$$

  - d^* = max $g(\lambda,\nu)$ finds best lower bound on p^*, obtained from Lagrange dual function
  - λ,ν, are dual feasible if $\lambda \geq 0$ and $\lambda,\nu \in \text{dom } g$
  - Convex optimization problem: since the objective to be maximized is concave and the constraint is convex

Weak and strong duality

- **Weak duality**: $d^* \leq p^*$
  - always holds (for convex and non-convex problems)
  - can be used to find nontrivial lower bounds for difficult problems
- **Strong duality**: $d^* = p^*$
  - does not hold in general
  - (usually) holds for convex problems
  - conditions that guarantee strong duality in convex problems are called *constraint qualifications*

Slater's constraint qualification

- **Strong duality** holds for a convex problem

$$\min f_0(x)$$
$$s.t.$$
$$f_i(x) \leq 0 \quad i = 1, \ldots, m$$
$$Ax = b$$

if it is *strictly* feasible, i.e.,

$$\exists x \in \mathcal{D} : f_i(x) < 0 (i = 1, \cdots, m); \quad Ax = b$$

  - It also guarantees that the dual optimum is attained when \`d^*>-∞, i.e., there exists a dual feasible (λ^* , ν^*) with g(λ^*;ν^*) = d^* = p^*

Complementary slackness

- When strong duality holds if x^* is *primal* optimal and (λ^*,ν^*) is *dual* optimal

$$
\begin{aligned}
f_0(x^*) = g(\lambda^*,\nu^*) &= \inf_{x\in\mathcal{D}}\left(f_0(x)+\sum_{i=1}^{m}\lambda_i^* f_i(x)+\sum_{i=1}^{p}\nu_i^* h_i(x)\right)\\
&\le f_0(x^*)+\sum_{i=1}^{m}\lambda_i^* f_i(x^*)+\sum_{i=1}^{p}\nu_i^* h_i(x^*)\\
&\le f_0(x^*)
\end{aligned}
$$

  - x^* minimizes $L(x,\lambda^*,\nu^*)$
  - $\lambda^* f_i(x^*)=0$ for $i=1,\cdots,m$, which implies one of the two

$$\lambda_i^* > 0 \Rightarrow f_i(x^*) = 0 \qquad\qquad f_i(x^*) < 0 \Rightarrow \lambda_i^* = 0$$

  - *Complementary*: either λ_i or $f_i(x)$ are zero
  - *Slack*: not binding

Karush-Kuhn-Tucker (KKT) conditions

- We now assume that the functions $f_0, \cdots, f_m$ and $h_0, \cdots, h_p$ are differentiable and $f_0(x^*) = g(\lambda^*, \nu^*)$. Then the following conditions hold

$$
\begin{aligned}
f_i(x^*) &\leq 0 \quad i = 1, \cdots, m \\
h_i(x^*) &= 0 \quad i = 1, \cdots, p \\
\lambda_i^* &\geq 0 \quad i = 1, \cdots, m \\
\lambda_i^* f_i(x^*) &= 0 \quad i = 1, \cdots, m \\
\nabla f_0(x^*) + \sum_{i=1}^{m} \lambda_i^* \nabla f_i(x^*) + \sum_{i=1}^{p} \nu_i^* \nabla h_i(x^*) &= 0
\end{aligned}
$$

KKT conditions for convex problems

- Single-cell single-user power allocation: allocate the power over the *N* channels with the goal of maximizing the rate subject to a

$$\max_{P} \sum_{n=1}^{N} \log_2(1 + \frac{P_n |H_n|^2}{\sigma^2})$$
$$s.t.$$
$$P_n \geq 0 \quad n = 1, \cdots, N$$
$$\sum_{n=1}^{N} P_n = P_0$$

- The problem is convex since it satisfies the Slater's qualifications

KKT conditions for convex problems

- In standard convex form ($\sigma^2_n = \sigma^2/|H_n|^2$)

$$\min_P -\sum_{n=1}^{N} \log_2(1+\tfrac{P_n}{\sigma_n^2})$$
$$s.t.$$
$$-P_n \le 0 \quad n=1,\cdots,N$$
$$\sum_{n=1}^{N} P_n = P_0$$

- The Lagrangian $L(P,\lambda,\nu)$ is

$$L(P,\lambda,\nu) = -\sum_{n=1}^{N} \log_2(1+\tfrac{P_n}{\sigma_n^2}) - \sum_{n=0}^{N} \lambda_n P_n + \nu\left(\sum_{n=1}^{N} P_n - P_0\right)$$

- The KKT conditions are

$$-P_n^* \le 0 \quad \lambda_n^* \ge 0 \quad \boxed{\lambda_n^* P_n^* = 0} \quad n=1,\cdots,N \quad \sum_{n=1}^{N} P_n^* = P_0$$

$$\boxed{-\frac{1}{\sigma_n^2+P_n^*}\frac{1}{\log 2} - \lambda_n^* + \nu^* = 0} \qquad n=1,\cdots,N$$

KKT conditions for convex problems

- The optimal value can be found using the *complementary slackness* condition

$$P_n^* \left(\nu^* \log 2 - \frac{1}{\sigma_n^2 + P_n^*} \right) = 0 \qquad n = 1, \cdots, N$$

- There are two possible solutions that depend on the value ν^*log2 …

$$P_n^* = \begin{cases} 0 & \nu^* \log 2 \geq 1/\sigma_n^2 \\ 1/(\nu^* \log 2) - \sigma_n^2 & \nu^* \log 2 < 1/\sigma_n^2 \end{cases} \qquad n = 1, \cdots, N$$

- …leading to the well-known waterfilling strategy

$$P_n^* = \max \left\{ 0, \left(\frac{1}{\log 2\nu^*} - \frac{\sigma^2}{G_n} \right) \right\} \quad n = 1, \cdots, N$$

Uniform power allocation

- Given any $\lambda \geq 0$ and ν, it holds

$$P_n + \sigma_n^2 = \frac{1}{\nu - \lambda_n} \frac{1}{\log 2} \quad n = 1, \cdots, N$$

and the duality gap $\Gamma(P,\lambda,\nu) = f_0(P) - g(\lambda,\nu)$ for any vector P is

$$\Gamma = \sum_{n=1}^{N} \left(\frac{\sigma_n^2}{P_n + \sigma_n^2} \frac{1}{\log 2} \right) + \nu P_0 - \frac{N}{\log 2}$$

after some manipulations one obtains

$$\Gamma = \frac{1}{\log 2} \sum_{n=1}^{N} \left(\frac{P_n}{\min_k \left\{ P_k + \sigma_k^2 \right\}} - \frac{P_n}{P_n + \sigma_n^2} \right)$$

Uniform power allocation

- If power is uniformly allocated on *M* channels it yields

$$
\begin{aligned}
\Gamma &= \frac{1}{\log 2}\sum_{n=1}^{M}\left(\frac{P_0/M}{P_0/M+\min_k\left\{\sigma_k^2\right\}}-\frac{P_0/M}{P_0/M+\sigma_n^2}\right)\\
&\leq \frac{1}{\log 2}\sum_{n=1}^{M}\frac{\sigma_n^2-\min_k\left\{\sigma_k^2\right\}}{P_0/M+\min_k\left\{\sigma_k^2\right\}+\sigma_n^2}
\end{aligned}
$$

- The strategy that allocates *uniform* power P_0/M over the *M* subchannels that would receive positive power in exact waterfilling is close to the optimum

Waterfilling dual: power minimization

- The power necessary to achieve a certain rate r_n over channel n is $P_n = (2^{r_n}-1)\cdot\sigma^2{}_n$
- Minimize the power with rate constraints

$$
\begin{gathered}
\min_{r} \sum_{n=1}^{N} (2^{r_n} - 1) \cdot \sigma_n^2 \\
s.t. \\
-r_n \leq 0 \quad n = 1, \cdots, N \\
\sum_{n=1}^{N} r_n = r_0
\end{gathered}
$$

Waterfilling dual: power minimization

- The Lagrangian is

$$L(r,\lambda,\nu) = \sum_{n=1}^{N} (2^{r_n}-1)\,\sigma_n^2 - \sum_{n=1}^{N} \lambda_n r_n - \nu\left(\sum_{n=1}^{N} r_n - r_0\right)$$

the optimal rate allocation is

$$r_n^* = \begin{cases} 0 & \nu^*/\log 2 \geq \sigma_n^2 \\ \log_2(\nu^*/\log 2) - \log_2 \sigma_n^2 & \nu^*/\log 2 < \sigma_n^2 \end{cases} \qquad n = 1,\cdots,N$$

and the power allocated on subcarrier *n* is

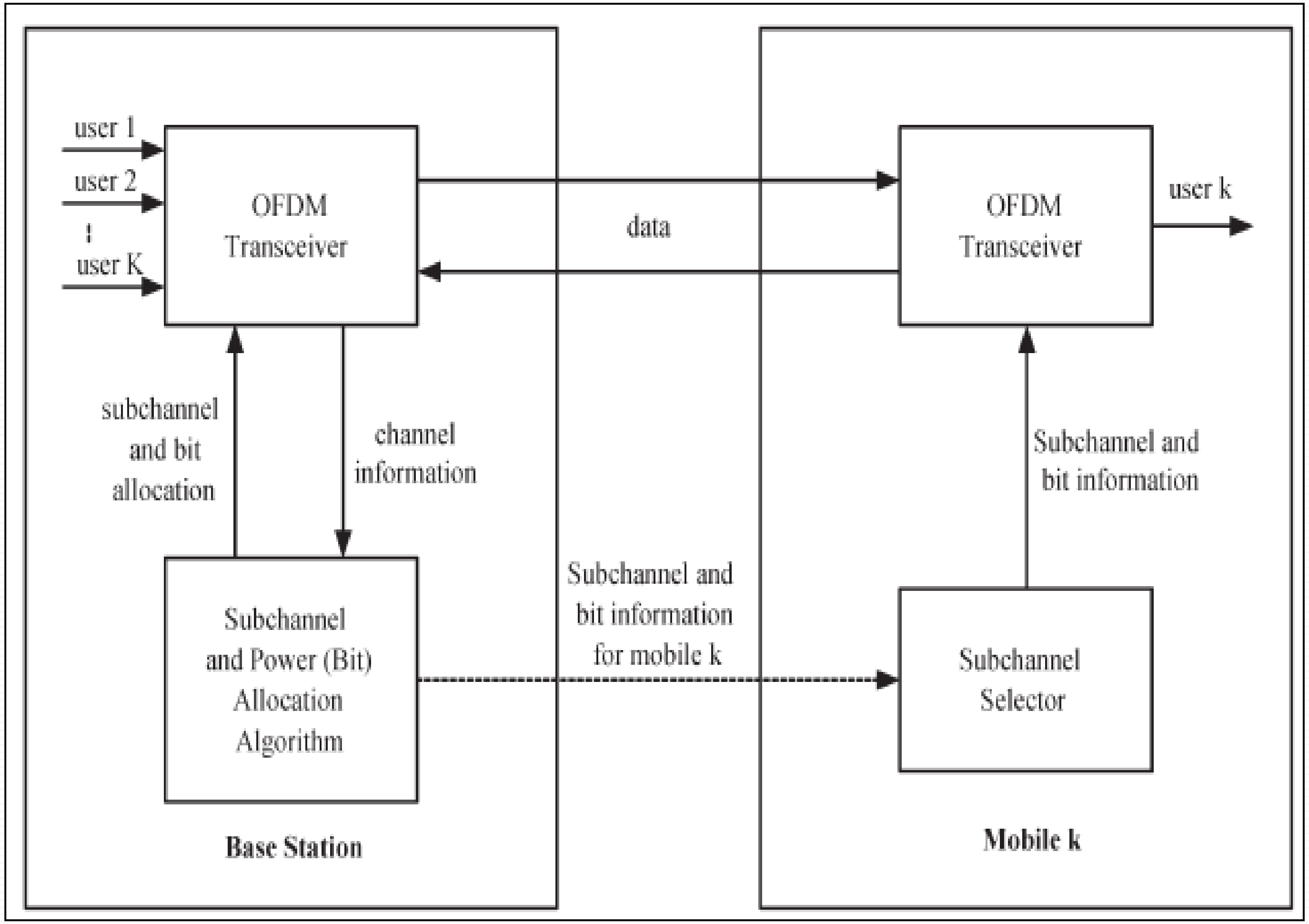
$$P_n = \left(\frac{\nu}{\log 2} - \sigma_n^2\right)^+$$

Single-cell allocation

OFDMA resource allocation

- The goal is to take advantage of the *multi-user* and *frequency* diversity of system to maximize the spectral efficiency and reduce the power consumption
- Radio resources are:
  - Transmission power
  - Transmission formats
  - Subcarriers
- As channels are statistically independent for each user, a channel that is “bad” for one user may be “good” for another

OFDMA resource allocation



Assumptions

- We consider the downlink of a wireless multi-carrier communication system with K users and N subcarriers
- Perfect CSI at the base station
- Ideal feedback channel to signal the assignment decision.
  - We introduce a binary allocation variable

$$\rho_{k,n} = \begin{cases} 1 & \text{if channel } n \text{ is assigned to user } k \\ 0 & \text{otherwise} \end{cases}$$

Resource allocation: constraints

- *Univocal* assignment:
  - Subcarriers are allocated univocally to the users: only one user at the time can occupy a given subcarrier

$$\sum_{k=1}^{K} \rho_{k,n} \leq 1 \quad n = 1,\ldots,N$$

  - The presence of an *integer* assignment variable greatly complicates the allocation problem since it makes the problem NOT convex

Resource allocation schemes

Rate adaptive

- *Objective*: maximize the overall rate r_{tot} subject to a global power constraint $\sum_n P_n = P_0$

$$r_{tot} = \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_{k,n} \log_2 \left(1 + \frac{P_n G_{k,n}}{\sigma^2} \right)$$

Margin adaptive

- *Objective*: minimize the overall power P_{tot} subject to the different users' rate constraints.

$$P_{tot} = \sum_{k=1}^{K} \sum_{n=1}^{N} P_n \rho_{k,n}$$

Rate adaptive schemes

Sum-rate maximization

- The *sum-rate maximization* problem is solved by
  1. assigning each sub-carrier to the user that maximizes its gain
  2. performing waterfilling over all the sub-carriers allocated.
- Such a solution maximizes the cell throughput but is *extremely unfair* since it privileges the users closest to the BS and starves all the others.

$$\max_{P,\rho} \sum_k \sum_n \log_2\left(1+\frac{P_n G_{k,n}}{\sigma^2}\right)\rho_{k,n}$$

$$s.t.$$

$$\sum_k \rho_{k,n} \le 1 \quad \forall n$$

$$\sum_n P_n \le P_0$$

$$\rho_{k,n} \in \{0,1\} \quad \forall k,n$$

Max-min rate allocation

- *Fairness* is introduced by allocating resources with the goal of maximizing the minimum capacity offered to each user, thus introducing fairness among the users.
  - In general, fairness comes at the cost of a reduction of the overall throughput of the cell.

$$\max_{P,\rho} \min_{k} \sum_{n} \log_2\left(1+\frac{P_n G_{k,n}}{\sigma^2}\right)\rho_{k,n}$$

$$s.t.$$

$$\sum_{k} \rho_{k,n} \leq 1 \quad \forall n$$

$$\sum_{n} P_n \leq P_0$$

$$\rho_{k,n} \in \{0,1\} \quad \forall k,n$$

Max-min rate allocation

- Problem is NOT convex
- A heuristic solution is implemented:
  1. *Uniform* power allocation on all sub-channels $P=P_0/N$
  2. A greedy *assignment* strategy that iteratively allocates the sub-carriers to the user with the smallest rate

1) *Initialization*

Set $r(k)=0$; $\Omega_k=\emptyset$ for all $k=1,\cdots,K$;

$\mathcal{A}=\{1,2,\cdots,N\}$ and $P=P_0/N$

2) *Assign at least one subcarrier to each user*

for $k=1$ to K

a) Find $\hat{n}=\arg\max_{\tilde{n}\in\mathcal{A}} G_{k,\tilde{n}}$

b) Update $r(k)$, Ω_k and $\mathcal{A}$

$r(k)=\log_2\left(1+PG_{k,\hat{n}}/\sigma^2\right)$;

$\Omega_k=\Omega_k\cup\{\hat{n}\}$; $\mathcal{A}=\mathcal{A}-\{\hat{n}\}$

3) *Distribute the remaining subcarriers*

while $\mathcal{A}\neq\emptyset$

a) Find $\hat{k}=\arg\min_{\tilde{k}} r(\tilde{k})$

b) Find $\hat{n}=\arg\min_{\tilde{n}\in\mathcal{A}} G_{\hat{k},\tilde{n}}$

c) Update $r(\hat{k})$ and $\mathcal{A}$

$r(\hat{k})=r(\hat{k})+\log_2\left(1+PG_{\hat{k},\hat{n}}/\sigma^2\right)$;

$\Omega_{\hat{k}}=\Omega_{\hat{k}}\cup\{\hat{n}\}$; $\mathcal{A}=\mathcal{A}-\{\hat{n}\}$

Sum-rate maximization with proportional rate constraints

- Different users may require different data rates. In this case, a fair solution is to allocate radio resources proportionally to the users' different rate constraints.

$$\max_{P,\rho} \sum_k \sum_n \log_2\left(1+\frac{P_n G_{k,n}}{\sigma^2}\right)\rho_{k,n}$$

$$s.t.$$

$$\sum_k \rho_{k,n} \le 1 \quad \forall n$$

$$\sum_n P_n \le P_0$$

$$r_1 : r_2 : \ldots : r_K = \gamma_1 : \gamma_2 : \ldots : \gamma_K$$

$$\rho_{k,n} \in \{0,1\} \quad \forall k,n$$

Sum rate maximization with proportional rate constraints

- The optimization problem is a *mixed binary integer programming* problem and as such is not convex and in general very hard to solve. We follow an heuristic approach:
  - *Subcarrier allocation phase*: assuming an uniform power distribution, the subcarriers are allocated complying as much as possible with the proportional rate constraints.
  - *Power allocation phase*: once the subcarrier are allocated to the users, the power is distributed so that the proportional rate constraints are exactly met.

Sum rate maximization with proportional rate constraints: subcarrier allocation

1) *Initialization*

Set $r(k) = 0$; $\Omega_k = \emptyset$ for all $k = 1, \cdots, K$;

$\mathcal{A} = \{1, 2, \cdots, N\}$ and $P = P_0/N$

2) *Assign at least one subcarrier to each user*

for $k = 1$ to K

a) Find $\hat{n} = \arg\max_{\tilde{n} \in \mathcal{A}} G_{k,\tilde{n}}$

b) Update $r(k)$, Ω_k and $\mathcal{A}$

$r(k) = \log_2 \left(1 + PG_{k,\hat{n}}/\sigma^2\right)$;

$\Omega_k = \Omega_k \cup \{\hat{n}\}$; $\mathcal{A} = \mathcal{A} - \{\hat{n}\}$

3) *Distribute the remaining subcarriers*

while $\mathcal{A} \neq \emptyset$

a) Find $\hat{k} = \arg\min_{\tilde{k}} \frac{r(\tilde{k})}{\gamma_{\tilde{k}}}$

b) Find $\hat{n} = \arg\max_{\tilde{n} \in \mathcal{A}} G_{\hat{k},\tilde{n}}$

c) Update $r(\hat{k})$ and $\mathcal{A}$

$r(\hat{k}) = r(\hat{k}) + \log_2 \left(1 + PG_{\hat{k},\hat{n}}/\sigma^2\right)$;

$\Omega_{\hat{k}} = \Omega_{\hat{k}} \cup \{\hat{n}\}$; $\mathcal{A} = \mathcal{A} - \{\hat{n}\}$

- At each iteration, the user with the lowest proportional capacity has the option to pick the best sub-channel.
- The sub-channel allocation algorithm is suboptimal, because it is greedy and assumes a uniform distribution of power.

Sum rate maximization with proportional rate constraints: power allocation

- The proportional rate constraints are enforced by allocating the power to the users in two steps
  1. Find for each user the expression of $P_k^{(tot)}$ the total power allocated to each user k.
  2. Distribute the power among users in such a way that the proportional rate constraints are met and the overall power does not exceed P_0

Sum rate maximization with proportional rate constraints: power allocation

- The Lagrangian is ($Z_{k,n}=G_{k,n}/\sigma^2$) :

$$
\begin{aligned}
L(P,\lambda) &= \sum_{k=1}^{K}\sum_{n\in\Omega_k}\log_2\left(1+P_{k,n}Z_{k,n}\right)+\lambda_1\left(\sum_{k=1}^{K}\sum_{n\in\Omega_k}P_{k,n}-P_0\right)\\
&+\sum_{k=2}^{K}\lambda_k\left(\sum_{n\in\Omega_1}\log_2\left(1+P_{1,n}Z_{1,n}\right)-\frac{\gamma_1}{\gamma_k}\sum_{n\in\Omega_k}\log_2\left(1+P_{k,n}Z_{k,n}\right)\right)
\end{aligned}
$$

- Leading to the power per user $P_k^{(tot)}$

$$P_k^{tot} = N_k P_{k,1} + \sum_{n=1}^{N_k}\frac{Z_{k,n}-Z_{k,1}}{Z_{k,n}Z_{k,1}}$$

- The optimal distribution of the $P_k^{(tot)}$ is found iteratively with the Newton-Raphson method.

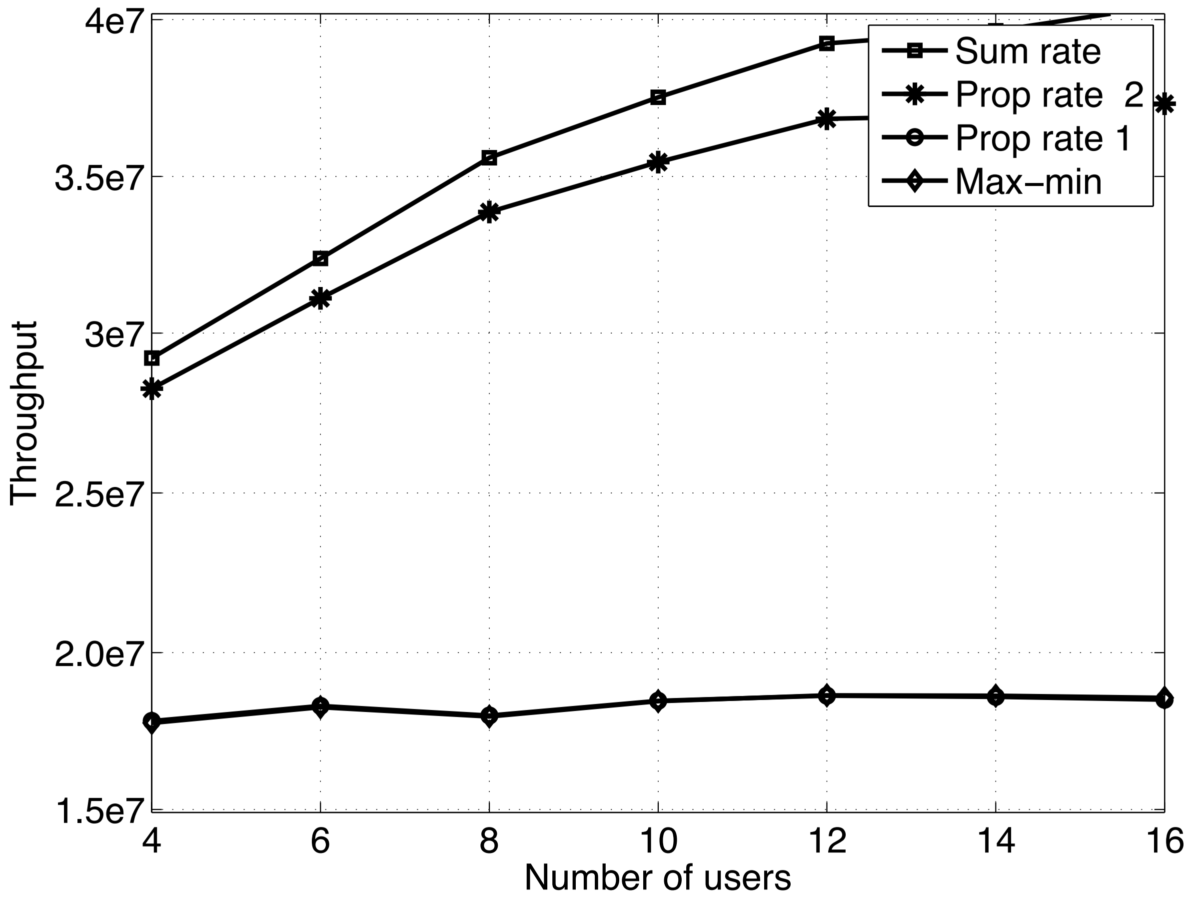
Rate adaptive results

- **Simulation parameters**
  - Number of cells: 1
  - Maximum BS transmission power: 1 W
  - Cell radius: 500 m
  - MT speed: static
  - Carrier frequency: 2 GHz
  - Number of sub-carriers: 192
  - Sub-carrier bandwidth 15 kHz
  - Path loss exponent: 4
  - Log-normal shadowing standard dev. 8 dB
  - Small-scale fading Typical Urban (TU)

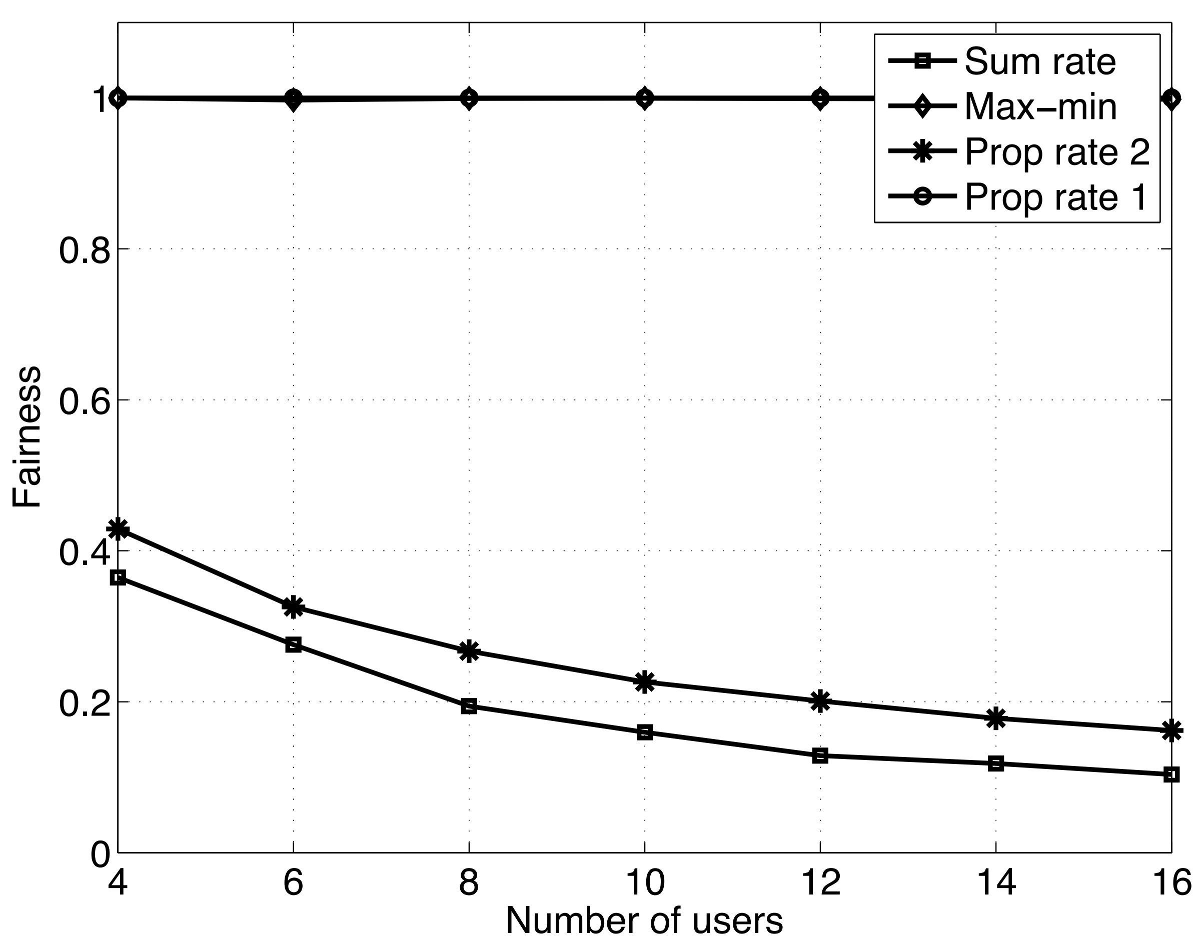
Rate adaptive results

- Algorithms simulated
  - Sum rate maximization
  - Max-min
  - Sum rate maximization with proportional rate constraints
    - Equal rate constraints $\gamma_1 = \gamma_2 = \ldots = \gamma_K$
    - Rate constraints proportional to the pathloss

Rate adaptive results



Rate adaptive results



Margin adaptive schemes

Resource allocation: constraints

- Each user has to meet a certain target rate, which constraints the task of resource allocation

$$r(k) = \sum_{n=1}^{N} r_{k,n} \quad k = 1,\ldots,K$$

  - The power necessary for user k to transmit with rate $r_{k,n}$ on subcarrier n is

$$P_{k,n} = \left(2^{r_{k,n}} - 1\right) / Z_{k,n}$$

WCLM: formulation

- The allocator solves the optimization problem by assigning the subcarriers and the rate on each subcarrier.
- Address the fairness issue but there are no explicit limits to the transmitted power.
- Problem is NOT convex

$$\min_{r,\rho} \sum_k \sum_n \left(2^{r_{k,n}} - 1\right) \frac{\rho_{k,n}}{Z_{k,n}}$$

$$s.t.$$

$$\sum_k \rho_{k,n} \le 1 \quad \forall n$$

$$\sum_n r_{k,n}\rho_{k,n} \ge r(k)$$

$$\rho_{k,n} \in \{0,1\} \quad \forall k,n$$

WCLM: convexification

1. Relax the integer allocation variable $\rho_{k,n}$.
   - Another way to interpret the optimization is to consider $\rho_{k,n}$ as the time-sharing factor for the k user of subcarrier n.
2. Introduce a new rate variable $s_{k,n}=r_{k,n}\rho_{k,n}$ so that the objective function becomes convex (positive semidefinite Hessian)

$$\min_{s,\rho} \sum_{n=1}^{N} \sum_{k=1}^{K} \rho_{k,n} \left(2^{\frac{s_{k,n}}{\rho_{k,n}}} - 1 \right) \frac{1}{Z_{k,n}}$$

$$s.t.$$

$$\sum_{n=1}^{N} s_{k,n} = r(k)$$

$$\sum_{k=1}^{K} \rho_{k,n} = 1$$

$$\rho_{k,n} \in [0,1]$$

3. Once the problem is convex, it can be solved in the *dual domain*

WCLM dual function

- Lagrangian of the problem is

$$L(s,\rho,\lambda,\mu)=\sum_{n=1}^{N}\sum_{k=1}^{K}\rho_{k,n}\left(2^{\frac{s_{k,n}}{\rho_{k,n}}}-1\right)\frac{1}{Z_{k,n}}-\sum_{k=1}^{K}\lambda_k\left(\sum_{n=1}^{N}s_{k,n}-r(k)\right)-\sum_{n=1}^{N}\mu_n\left(\sum_{k=1}^{K}\rho_{k,n}-1\right)$$

  - Given the Lagrangian multipliers λ yields

$$s_{k,n}=\begin{cases}0 & \lambda_k\le \log 2/Z_{k,n}\\ \rho_{k,n}\log_2\left(\lambda_k Z_{k,n}/\log 2\right) & \lambda_k>\log 2/Z_{k,n}\end{cases}$$

  - While for $\rho_{k,n}$ holds

$$\rho_{k,n}=\begin{cases}1 & \text{if } k=\arg\min \frac{1}{Z_{k,n}}\left(\frac{\lambda_k Z_{k,n}}{\log 2}-1\right)-\lambda_k\log_2\left(\frac{\lambda_k Z_{k,n}}{\log 2}\right)\\ 0 & \text{else}\end{cases}$$

WCLM: algorithm flow chart

- The algorithm is iterative
  - Start from some arbitrary values of λ and computes $\rho_{k,n}$ and $s_{k,n}$.
  - If the users' rate constrains are not satisfied iteratively increase the values of λ until all the rate constraints are met. This procedure requires the inversion of non –linear functions to converge.

1) *Initialization*

- for $k = 1$ to K

  Set λ_k to an aritrary small value;

  Compute $s_{k,n}/\rho_{k,n}$ for all subcarriers
- for $n = 1 : N$

  Assign subcarrier n to the user k that minimizes $H_{k,n}(\lambda_k)$;

2) *Iterative algorithm*

while $\exists k | \sum_n s_{k,n} < r(k)$

a) Find $\hat{k} = \arg\max_{\tilde{k}} \left(r(\tilde{k}) - \sum_n s_{\tilde{k},n} \right)$

b) Increment $\lambda_{\hat{k}}$ so that $\sum_n s_{\hat{k},n} = r(\hat{k})$

c) Update the number of subcarriers allocated to user $\hat{k}$

d) Update the rate assigned to all users computing $\sum_n s_{k,n} \quad \forall k$

WCLM algorithm

- Complexity is a major issue: due to the nature of the allocation problem, the solution proposed is *iterative*: depending on system parameters, convergence may be extremely slow.
- Solution admits non-integer values of the allocation variable.
  - It is necessary to implement a heuristic (multi-user adaptive OFDM, MAO) to set the vector of allocation variables to integer values

Linear programming

- Resource allocation can be formulated as a linear programming problem:
  - Linear objective function
  - Linear constraints

Linear programming: multiple tx formats

- All rate requests are expressed as a multiple integer of a certain fixed rate corresponding to a *spectral efficiency* η

$$\eta = \log_2\left(1 + P_{k,n} Z_{k,n}\right)$$

- The power necessary for user k to transmit the rate $b\eta$ ($b=0,\cdots,B$) on the subcarrier n is a *fixed cost*

$$P_{k,n}^{(b)} = (2^{b\eta} - 1) / Z_{k,n}$$

Linear programming: multiple tx formats

- After linearization of the objective function and of the constraints, resource allocation can be formulated as a linear integer programming (LIP) problem
- Combinatorial problem with exponential complexity in *N*, *K*, and *B*

$$\min_{\rho} \sum_k \sum_n \sum_b P_{k,n}^{(b)} \rho_{k,n}^{(b)}$$

$$s.t.$$

$$\sum_k \sum_b \rho_{k,n}^{(b)} \leq 1 \qquad n = 1,\ldots,N$$

$$\sum_k \sum_b b\eta\rho_{k,n}^{(b)} = r(k) \qquad k = 1,\ldots,K$$

$$\rho_{k,n}^{(b)} \in \{0,1\} \quad \forall b,k,n$$

Linear programming: single tx format

- Provided that there is enough multi-user diversity, it is possible adopt only one transmission format (B=1) with very limited performance loss.
- The rate requirements $r(k)$ are translated into a minimum number $n(k)$ of subcarriers to be allocated per user
- By relaxing the integer constraint, RRA turns into a standard LP problem

$$\min_{\rho} \sum_k \sum_n P_{k,n} \rho_{k,n}$$

$$s.t.$$

$$\sum_k \rho_{k,n} \le 1 \qquad n = 1,\ldots,N$$

$$\sum_k \rho_{k,n} = n(k) \qquad k = 1,\ldots,K$$

$$\rho_{k,n} \in \{0,1\}$$

Linear programming: single tx format

- The relaxed LP RRA problem has the characteristic that it can be modeled as a a *network flow* problem.
  - The *network simplex method* (NSM) is the most efficient solver for min-cost-max-flow network problems and outperforms other existing techniques
  - Because of its topology, the solution of the relaxed LP RRA is *integral* and thus, regardless of the relaxation, always a combination of 0 and 1
- The single format choice allows a great simplification of the solution of the RRA problem at the cost of only a modest worsening of system performance. Dynamical assignment of subcarriers already provides a great deal of diversity!

Linear programming: power allocation

- After having solved the LP single-format resource allocation, power can be further reduced by solving a single-user waterfilling problem for each user on the assigned subcarriers.
- For user k, who is allocated the set Ω_k of subcarriers, the problem is formulated as

$$\min_{r} \sum_{n \in \Omega_k} \left(2^{r_{k,n}} - 1\right) \frac{1}{Z_{k,n}}$$

$$s.t.$$

$$\sum_{n \in \Omega_k} r_{k,n} = r(k)$$

Optimal allocation

- The solution of an optimization problem can be bounded by resorting to the Lagrange dual
- Duality gap Γ is the difference between the solution of the primal problem and the solution of the dual problem
- *Qualification conditions*. It has been showed that in multi-carrier applications, even if the original RRA problem is *non-convex*, the duality gap tends to zero as the number of tones goes to infinity.

Optimal rate allocation: primal

- The primal problem is formulated as a minimization problem with standard rate and exclusive allocation constraints
- The problem is combinatorial (i.e. all possible allocations should be evaluated!) and its complexity grows exponentially with *K* and *N*

$$\min_{\boldsymbol{\rho},\mathbf{r}} \sum_{n=1}^{N}\sum_{k=1}^{K}\left(2^{r_{k,n}}-1\right)\frac{\rho_{k,n}}{Z_{k,n}}$$

$$s.t.$$

$$\sum_{n=1}^{N} r_{k,n}\rho_{k,n} \geq r(k)\ \forall k$$

$$\Re = \left\{\rho_{k,n} \mid \sum_{k=1}^{K}\rho_{k,n}=1;\ \rho_{k,n}\in\{0,1\}\right\}$$

Optimal allocation: Lagrange dual

- The Lagrangian is

$$L(r,\rho,\lambda)=\sum_{n=1}^{N}\sum_{k=1}^{K}\left(2^{r_{k,n}}-1\right)\frac{\rho_{k,n}}{Z_{k,n}}-\sum_{n=1}^{N}\lambda_k\left(\sum_{n=1}^{N}r_{k,n}\rho_{k,n}-r(k)\right)$$

defined over the set $\mathcal{R}$ and all the positive rates $r_{k,n}$

- The Lagrangian dual function is

$$g(\boldsymbol{\lambda})=\min_{\mathbf{r},\boldsymbol{\rho}}\sum_{n=1}^{N}\left(2^{r_{k,n}}-1\right)\frac{\rho_{k,n}}{Z_{k,n}}-\sum_{k=1}^{K}\lambda_k\left(\sum_{n=1}^{N}r_{k,n}\rho_{k,n}-r(k)\right)$$
$$s.t.$$
$$\sum_{k=1}^{K}\rho_{k,n}=1\quad\forall n$$

Optimal allocation: Lagrange dual

- The Lagrange dual of the RRA can be written as the sum of *N* reduced-complexity minimization problems

$$g_n(\lambda) = \min_{r,\rho} \sum_{k=1}^{K} \left(\left(2^{r_{k,n}} - 1\right) \frac{\rho_{k,n}}{Z_{k,n}} - \lambda_k r_{k,n} \right)$$

$$s.t.$$

$$\sum_{k=1}^{K} \rho_{k,n} = 1 \quad \forall n$$

- Solving the per-carrier problem still involves an exhaustive search over the whole set of users.

Optimal allocation: dual update method

- The solution to the Lagrange dual problem is found following an *iterative* process:
  - Given the multiplier vector $\boldsymbol{\lambda}$, we find $g(\boldsymbol{\lambda})$ and the rates allocated for each user
  - The rate results for the different users contribute to the subgradient

$$d(k) = r(k) - \sum_{n=1}^{N} r_{k,n}(\boldsymbol{\lambda}) \quad k = 1,\cdots,K$$

  - The subgradient is employed to update the multiplier vector $\boldsymbol{\lambda}$ (*Ellipsoid method*)

Optimal allocation: ellipsoid method

- It is the multi-dimenional extension of the bisection method
- The idea is to localize the set of candidate λ s within some closed and bounded set.
- Then, by evaluating the subgradient of $g(\lambda)$ at an appropriately chosen center of such a region, roughly half of the region may be eliminated from the candidate set.
- The iterations continue as the size of the candidate set diminishes until it converges to an optimal

- An ellipsoid with a center z and a shape defined by positive semidefinite matrix **A** is defined as

$$E(A, z) = \{x | (x-z)^T A(x-z) \leq 1\}$$

- The update rule is the following

1) $\tilde{\mathbf{d}}_\mathbf{i} = \dfrac{\mathbf{d}_\mathbf{i}}{\sqrt{\mathbf{d}_\mathbf{i}{}^T \mathbf{A}_\mathbf{i}{}^{-1} \mathbf{d}_\mathbf{i}}}$

2) $\mathbf{z}_{\mathbf{i+1}} = \mathbf{z}_\mathbf{i} - \dfrac{1}{K+1} \mathbf{A}_\mathbf{i}{}^{-1} \tilde{\mathbf{d}}_\mathbf{i}$

3) $\mathbf{A}_{\mathbf{i+1}}{}^{-1} = \dfrac{K^2}{K^2-1} \times \left(\mathbf{A}_\mathbf{i}{}^{-1} - \dfrac{2}{K+1} \mathbf{A}_\mathbf{i}{}^{-1} \tilde{\mathbf{d}}_\mathbf{i} \tilde{\mathbf{d}}_\mathbf{i}{}^T \mathbf{A}_\mathbf{i}{}^{-1} \right)$

Optimal power allocation

- The same approach can be used to solve the maximization of the sum of weighted rates

$$\max_{\rho,P} \sum_{k=1}^{K} w_k \sum_{n=1}^{N} \log_2\left(1 + P_n Z_{k,n}\right)\rho_{k,n}$$

$$s.t.$$

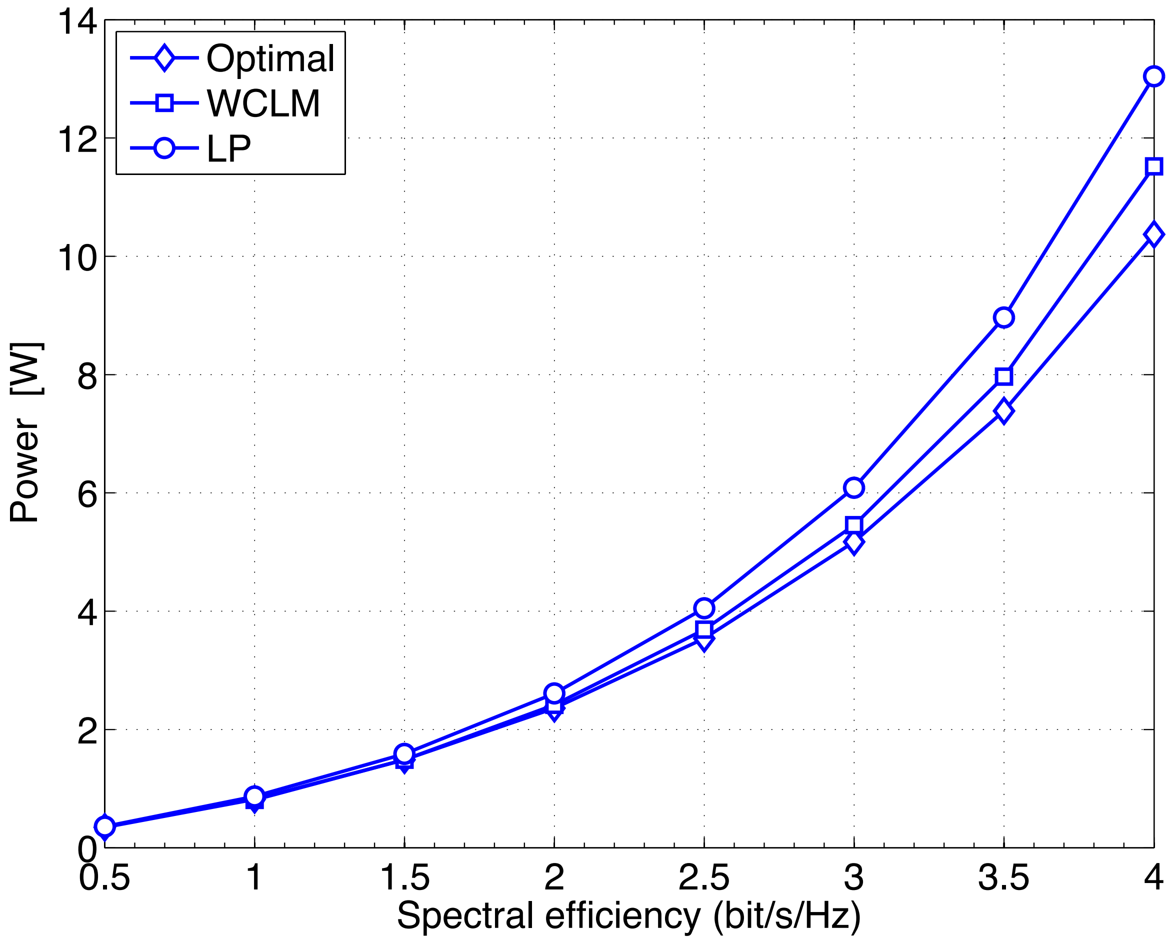
$$\sum_{n=1}^{N} P_n \le P_0$$

$$R = \left\{ \rho_{k,n} \mid \sum_{k=1}^{K} \rho_{k,n} = 1; \rho_{k,n} \in \{0,1\} \right\}$$

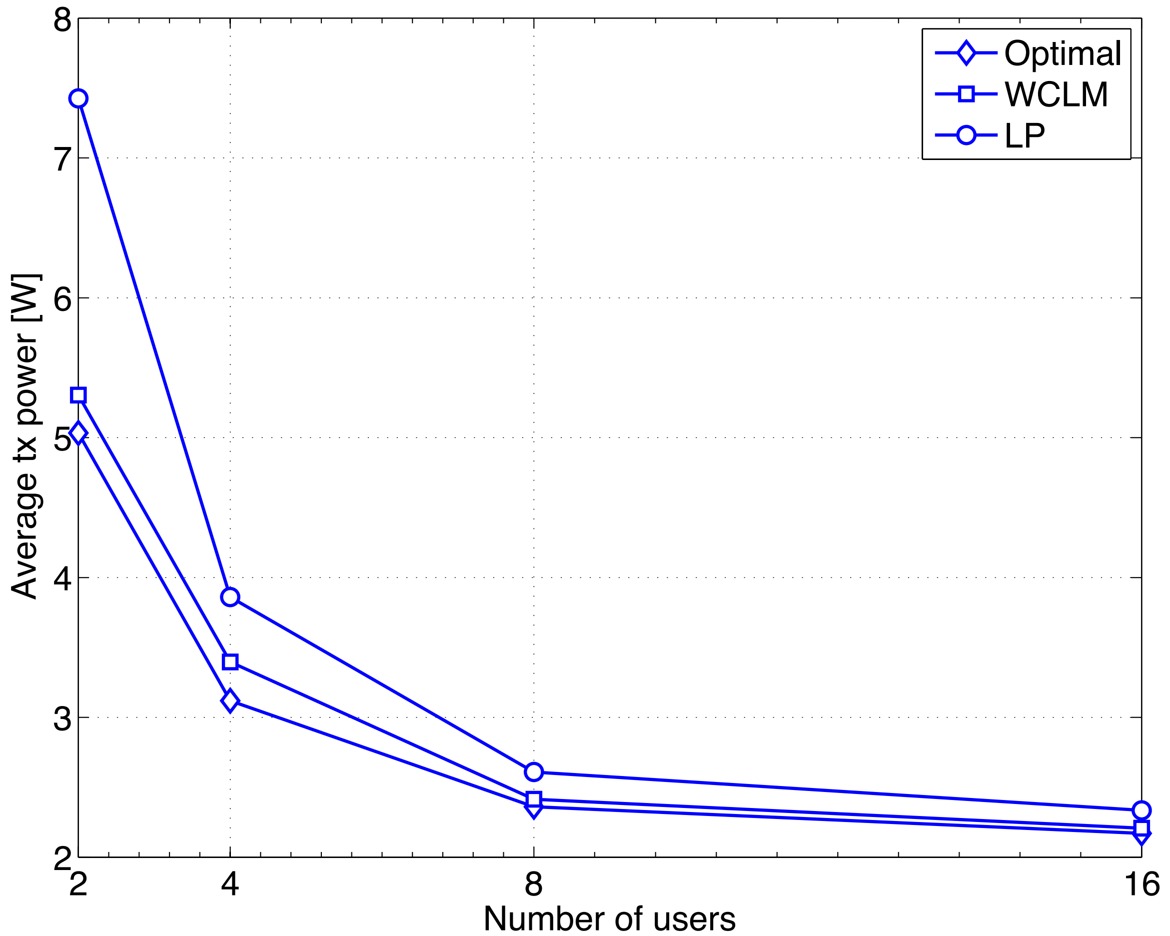
Simulation setup

- Number of cells = 1
- Radius of each cell R = 500 m
- Total available bandwidth W = 5MHz
- Center frequency = 2 GHz
- Number of subcarriers $N = 64$
- Number of users $K = 8$

Power vs. spectral efficiency



Power vs. number of users



MIMO resource allocation

MIMO system

- We are considering a $N_T \times N_R$ MIMO system with $N_T > N_R$ so that at least $Q = \lfloor N_T/N_R \rfloor$ users can transmit on the same frequency channel.
- Users signals are separated by the implementation of linear precoding and receving filters
- Signal model

$$\mathbf{z}_{k,n} = \mathbf{W}_{k,n}^H \mathbf{y}_{k,n} = \underbrace{\mathbf{W}_{k,n}^H \mathbf{H}_{k,n} \mathbf{B}_{k,n} \mathbf{s}_{k,n}}_{\text{Desired signal}} + \underbrace{\mathbf{W}_{k,n}^H \left(\mathbf{H}_{k,n} \sum_{j \neq k, j \in \mathcal{U}_n} \mathbf{x}_{j,n} + \eta_{k,n} \right)}_{\text{Multiple access interference and noise}}$$

MMO optimal allocation

- Margin adaptive optimization problem:
  - Optimize linear precoder, transmit power distribution and channel allocation to minimize the overall transmit power
  - Problem is NOT convex and prohibitively *complex*

$$\min_{\mathbf{x},\boldsymbol{\rho}} \sum_{n=1}^{N} \sum_{k \in \mathcal{U}_n} \operatorname{tr}\left(\mathbf{R}_{\mathbf{x}_{k,n}}\right)$$

$$s.t.$$

$$\sum_{n=1}^{N} \rho_{k,n} \log_2 \det\left(\mathbf{I}_{N_R} + \mathbf{H}_{k,n}\mathbf{R}_{\mathbf{x}_{k,n}}\mathbf{H}_{k,n}^{H}\mathbf{R}_{\mathbf{i}_{k,n}}^{-1}\right) \geq r(k) \quad 1 \leq k \leq K$$

$$\left|\mathcal{U}_n\right| \leq Q \quad 1 \leq n \leq N$$

Block diagonalization (BD) approach

- To simplify the problem, we first decide the precoding strategy and then allocate remaining resources (channels and power).
- By projecting each user's MIMO channel on the interference null space, users' channels are decoupled so that the users transmit on the same channel do not interfere with each other
- Allocation task is greatly simplified in absence of interference

BD-based RA

- Problem is still not convex but as the number of subcarriers increase the duality gap tends to zero and can be solved in the dual domain.

$$\min_{\mathbf{p},\boldsymbol{\rho}} \sum_{n=1}^{N} \sum_{k \in \mathcal{U}_n} P_{k,n}$$

$$s.t.$$

$$\sum_{n=1}^{N} \rho_{k,n} r_{k,n}\left(\mathbf{P},\boldsymbol{\rho}\right) \geq r(k) \quad 1 \leq k \leq K$$

$$\left|\mathcal{U}_n\right| \leq Q \quad 1 \leq n \leq N$$

$$P_{k,n} = \sum_{l=1}^{\ell_{k,n}} P_{k,n}^{(l)}$$

$$r_{k,n}\left(\mathbf{P},\boldsymbol{\rho}\right) = \sum_{l=1}^{\ell_{k,n}} \log_2\left(1 + \frac{P_{k,n}^{(l)} g_{k,n}^{(l)}}{\sigma_0^2}\right)$$

BD-based RA – Dual domain

- On each subcarrier the dual function must be evaluated over all the possible combinations of Q users
- For each combination the precoding and receive linear filters must be evaluated!!

$$g_v(\boldsymbol{\mu}) = \min_{\mathbf{P},\boldsymbol{\rho}} \sum_{k \in \mathcal{U}_v} P_{k,v} - \mu_k r_{k,v}(\mathbf{P},\boldsymbol{\rho})$$
$$s.t.$$
$$|\mathcal{U}_v| \leq Q$$

1. Exhaustively compute all user combination
2. For each user combination evaluate

$$P_{k,n}^{(l)} = \left(\frac{\mu_k}{\log 2} - \frac{\sigma_0^2}{g_{k,n}^{(l)}(\boldsymbol{\rho})} \right)^+$$

3. Choose the combination that minimizes the metric

Successive channel assignment

- To reduce allocation complexity, we first group the users on the base of their channel quality and then sequentially solve the RA problem.
- The implementation of a sequential allocation strategy forces a change in the design of the linear precoder.
  - The users of a group do not interfere with the users already allocated but do generate interference versus the sets of user allocated successively.
  - MAI is treated as *spatially colored noise*

Successive channel assignment

- Taking into account the colored interference the allocation problem can be formulated as the solution of Q successive problems

$$\min_{\mathbf{x},\boldsymbol{\rho}} \sum_{k\in\mathcal{K}_q} \sum_{n=1}^{N} \operatorname{tr}\left(\mathbf{R}_{\mathbf{x}_{k,n}}\right)$$

$$s.t.$$

$$\sum_{n=1}^{N} \rho_{k,n} \log_2 \det\left(\mathbf{I}_{N_R} + \overline{\mathbf{H}}_{k,n}^{(q)} \mathbf{R}_{\mathbf{x}_{k,n}} \overline{\mathbf{H}}_{k,n}^{(q)H} \mathbf{R}_{\mathbf{i}_{k,n}}^{-1}\right) \geq r(k) \quad k\in\mathcal{K}_q$$

$$\sum_{k\in\mathcal{K}_q} \rho_{k,n} \leq 1 \quad 1 \leq n \leq N$$

Successive channel assignment

- Problem becomes more tractable by whithening the colored noise at the receiver multiplying the received signal by $\mathbf{R}_{\mathbf{i}_{k,n}}^{-1/2}$

$$\min_{\mathbf{p},\mathbf{a}} \sum_{k\in\mathcal{K}_q} \sum_{n=1}^{N} \mathbf{P}_{k,n}$$

$$s.t.$$

$$\sum_{n=1}^{N} \rho_{k,n} r'_{k,n}\left(\mathbf{P},\boldsymbol{\rho}\right) \geq r(k) \quad k\in\mathcal{K}_q$$

$$\sum_{k\in\mathcal{K}_q} \rho_{k,n} \leq 1 \quad 1 \leq n \leq N$$

LP-based channel assignment

- Supposing that each user transmits with a fixed spectral efficiency on all his channels:
  - power needed becomes a cost
  - rate requirements $r(k)$ translates into requesting a certain number of channels $n(k)$
- Each successive problem can be formulated as a LP problem

$$\min_{\boldsymbol{\rho}} \sum_{n=1}^{N} \sum_{k\in\mathcal{K}_q} \rho_{k,n} P_{k,n}$$

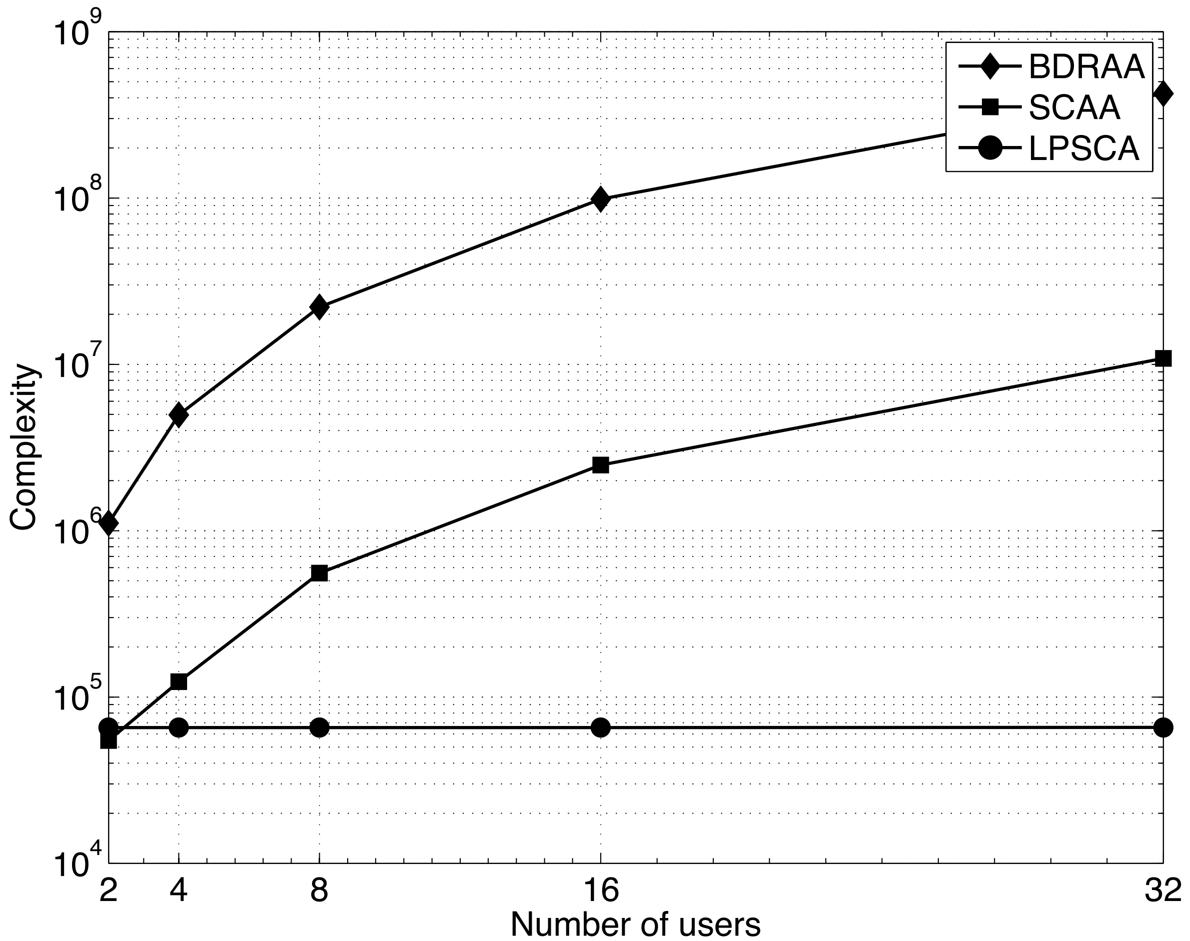
$$\sum_{n=1}^{N} \rho_{k,n} = n(k) \quad k \in \mathcal{K}_q$$

$$\sum_{k\in\mathcal{K}_q} \rho_{k,n} \leq 1 \quad n = 1,\ldots,N$$

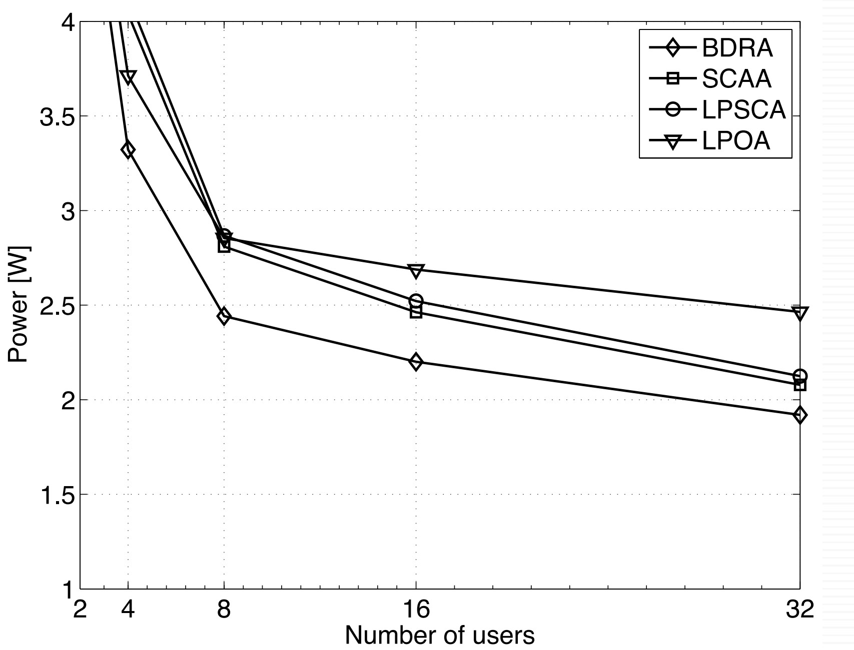
Simulation setup

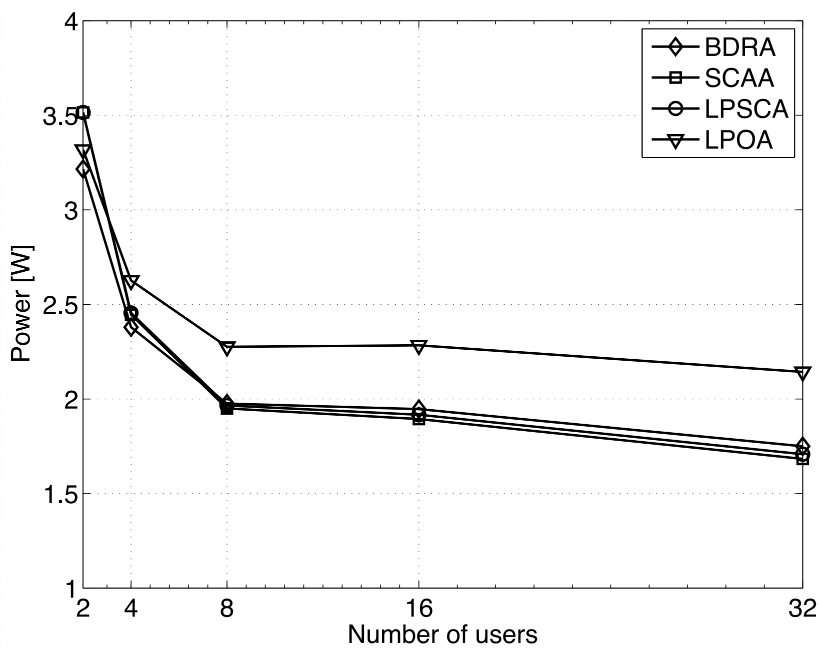
- Number of cells = 1
- Radius of each cell R = 500 m
- Total available bandwidth W = 5MHz
- Center frequency = 2 GHz
- Number of subcarriers $N = 64$
- Number of users $K = 8$
- Two scenarios: 4x2 and 2x1

Computational complexity

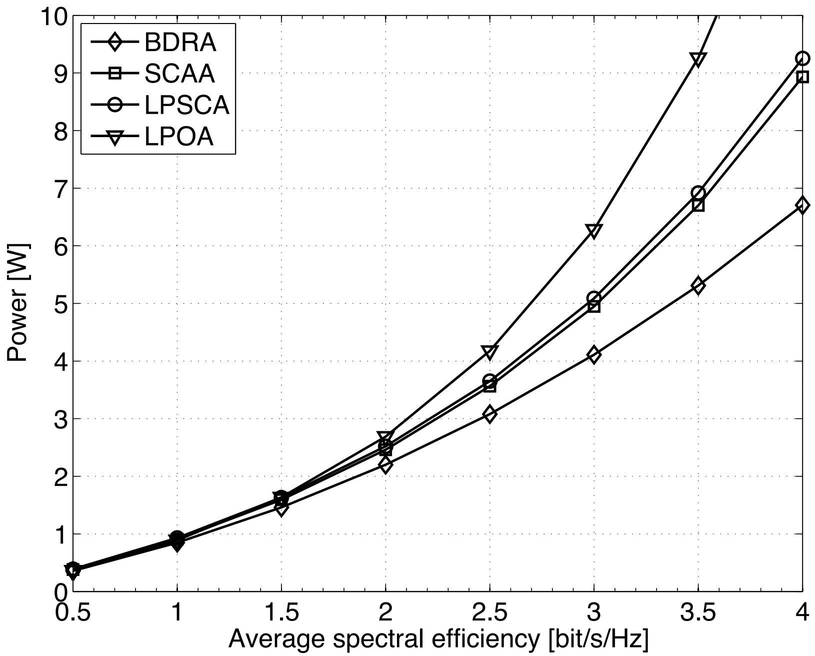


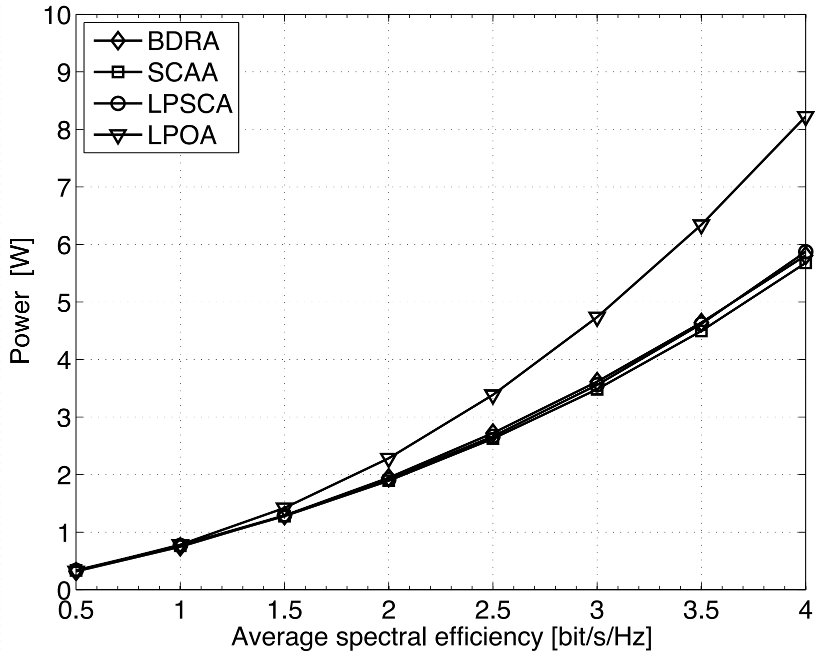
Power vs number of users





Power vs. spectral efficiency





Multi-cell algorithms

Multi-cellular RRA schemes

- We consider a downlink communication in an OFDMA-based multi-cell network with **full reuse** of the frequency spectrum among cells
- The most limiting factor for this systems is represented by multiple access interference (MAI), caused by users in adjacent cells that share the same spectrum

Multi-cell RRA: interference

- With respect to the conventional single-cell scenario, the main problem is the *feasibility* of the allocation.
  - Given a certain traffic configuration, there might be no solution that satisfies the rate requirements of all users in the system.
  - It is equivalent to the problem of power control for single-carrier cellular networks.
- RRA schemes need to enforce strategies designed to control the users' requirements in order to meet a feasible solution.

Multi-cell RRA: interference

- MAI depends on the users allocated on the same channel in other cells.
- Interference power is computed as

$$I_{k,n}^{(i)} = \sum_{j=1, j\neq i}^{N_{cells}} P_n^{(j)} G_{k,n,i}^{(j)}$$

- The required transmitted power to achieve a certain target ${\gamma_{k,n}}^{(i)}$ is:

$$P_{k,n}^{(i)} = \gamma_{k,n}^{(i)} \frac{\sigma^2 + I_{k,n}^{(i)}}{G_{k,n,i}^{(i)}}$$

Multi-cell power control

- Let us focus on subcarrier n
  - Suppose that in cell i it is allocated to user $k(i)$.
  - Let $G_{i,j} = G^{(j)}_{k(i),n,i}$, $\gamma^{(i)} = \gamma^{(i)}_{k,n}$ and $P^{(i)} = P^{(i)}_{n}$.
  - Power control consists in solving a set of linear equations in $\mathbf{P}$

$$
\begin{cases}
P^{(1)} = \gamma^{(1)}\left(\sigma^2 + \sum_{j=2}^{N_c} G_{1,j}P^{(j)}\right)\frac{1}{G_{1,1}} \\
\vdots \\
P^{(N_c)} = \gamma^{(N_c)}\left(\sigma^2 + \sum_{j=1}^{N_c-1} G_{N_c,j}P^{(j)}\right)\frac{1}{G_{N_c,N_c}}
\end{cases}
\Longrightarrow
\left(\mathbf{I} - \boldsymbol{\Gamma}\tilde{\mathbf{G}}\right)\mathbf{P} = \mathbf{u}
$$

$$u_i = \frac{\gamma_i \sigma^2}{G_i}; \left[\tilde{\mathbf{G}}\right]_{l,j} = \frac{G_{l,j}}{G_l}$$

$$\boldsymbol{\Gamma} = \mathrm{diag}\left(\gamma_1, \ldots, \gamma_{N_c}\right)$$

Multi-cell power control

- The matrix $\boldsymbol{\Gamma}\tilde{\mathbf{G}}$ has *non-negative* elements and it is by its nature irreducible. Invoking the Perron-Frobenius theorem, these three statements are equivalent
  - It exist a power vector $\mathbf{P}^*$: $\mathbf{P} = \left(\mathbf{I} - \boldsymbol{\Gamma}\tilde{\mathbf{G}}\right)^{-1} \mathbf{u}$
  - $\lambda_M\left(\boldsymbol{\Gamma}\tilde{\mathbf{G}}\right) < 1, \quad \lambda_M\left(\boldsymbol{\Gamma}\tilde{\mathbf{G}}\right)$ max eigenvalue of $\boldsymbol{\Gamma}\tilde{\mathbf{G}}$
  - $\lim_{k\to+\infty}\left(\Gamma\tilde{\mathbf{G}}\right)^k = 0$

Multi-cellular RRA schemes

- In a multi-cell system, RRA algorithms can be classified as *distributed* and *centralized*
  - In a distributed scheme, resource allocation is performed locally by each base station, exploiting the knowledge of channel conditions of only the users in the cell
  - In a centralized approach, a radio network controller (*god*), that ideally knows the channel state information of all users in the system, assigns the radio resources aiming at a global optimum

Distributed schemes

- Attractive because of the limited (!!) amount of feedback and computational complexity.
- Each cell has its own controller: RRA is performed on the base of the information available in the cell.
- Hybrid schemes allow a certain amount of information exchanged on the network backbone.
- The lack of centralized information is partially compensated by the implementation of iterative algorithms.

Centralized schemes

- Centralized solutions aim at optimizing the system performance globally:
  - Controller possesses full information about all users in the system
- Unfortunately, they are practically unfeasible due to
  - the large amount of signaling they require
  - their complexity, which grows exponentially with the number of users in the system (scale with the number of cells)

Multi-cell algorithms: distributed schemes

Distributed schemes: the PB algorithm

- This distributed algorithm addresses the problem by dividing it in three steps.
  1. Set max SINR bounds per subcarrier per user per cell
  2. Solve the allocation problem
  3. Implement an admission control strategy

PB algorithm: SINR bounds

- The spectral radius of matrix $\lambda_M\left(\mathbf{\Gamma}\tilde{\mathbf{G}}\right)$ is lower than any sub-multiplicative matrix norm of $\mathbf{\Gamma}\tilde{\mathbf{G}}$

$$\lambda_M\left(\mathbf{\Gamma}\tilde{\mathbf{G}}\right) \le \left\|\mathbf{\Gamma}\tilde{\mathbf{G}}\right\|_\infty = \max_{1\le i\le N_c}\left(\frac{\gamma_{k,n}^{(i)}\sum_{j,j\ne i} G_{k,n,i}^{(j)}}{G_{k,n,i}^{(i)}}\right)$$

- By imposing $\lambda_M\left(\mathbf{\Gamma}\tilde{\mathbf{G}}\right) < 1$, we can set a bound for the max target SINR per user per subcarrier, i.e.

$$\frac{\gamma_{k,n}^{(i)}\sum_{j,j\ne i} G_{k,n,i}^{(j)}}{G_{k,n,i}^{(i)}} < 1 \quad \Rightarrow \quad \gamma_{k,n}^{(i)} < \frac{G_{k,n,i}^{(i)}}{\sum_{j,j\ne i} G_{k,n,i}^{(j)}} = E_{k,n}^{(i)}$$

PB algorithm: the allocation problem

- PB propose an iterative scheme that
  1. implements a heuristic that allocates the subcarriers to users
  2. solves a convex problem in the SINR variable designed to minimize the transmit power, having assumed that the interference power is fixed
  3. performs power control so that each user meets its target SINR
- Algorithm is iterated until convergence

$$\min_{\gamma} \gamma_{k,n}^{(i)} \frac{I_{k,n}^{(i)} + \sigma^2}{G_{k,n,i}^{(i)}}$$

$$s.t.$$

$$\sum_{n=1}^{N} \log_2(1+\gamma_{k,n}^{(i)}) \geq r(k) \quad \forall k$$

$$\gamma_{k,n}^{(i)} \leq E_{k,n}^{(i)} \quad \forall k,n$$

PB algorithm: the allocation problem

- The admission control strategy consists in switching off those users that:
  - due the max SINR bounds, do not reach their target rates
  - exceed a certain predetermined power limit
- There is a fairness problem!!

Distributed schemes: a LP approach

- As in the single-cell scenario, we have formulated a linear programming approach with just a single transmission format for all users.
- The distributed approach leads to an iterative procedure: at the beginning of each new iteration the resources' costs in each cell are updated taking into account the interference levels of the previous iteration.
- Due to interference, allocation in one cell perturbs the allocation in all neighboring cells

LP algorithm: load control

- Allocation convergence is not guaranteed and the algorithm needs to be modified to reach a stable allocation.
- We implement a load control mechanism that progressively reduce the total amount of resources allocated in each cell until a stable allocation is achieved
- LP formulation still maintains the network flow topology

LP algorithm: formulation

- In each cell the LP problem is formulated so that a certain number $N^{(i)}$ of subcarriers has to be allocated.
- Each user k can get *at most* $n^{(i)}(k)$ resources.

$$\min_{\rho} \sum_{k=1}^{K} \sum_{n=1}^{N} P_{k,n}^{(i)} \rho_{k,n}^{(i)}$$

$$s.t.$$

$$\sum_{k=1}^{K} \rho_{k,n}^{(i)} \leq 1 \qquad \forall n$$

$$\sum_{n=1}^{N} \rho_{k,n}^{(i)} \leq n^{(i)}(k) \qquad \forall k$$

$$\sum_{k=1}^{K} \sum_{n=1}^{N} \rho_{k,n}^{(i)} = N^{(i)}$$

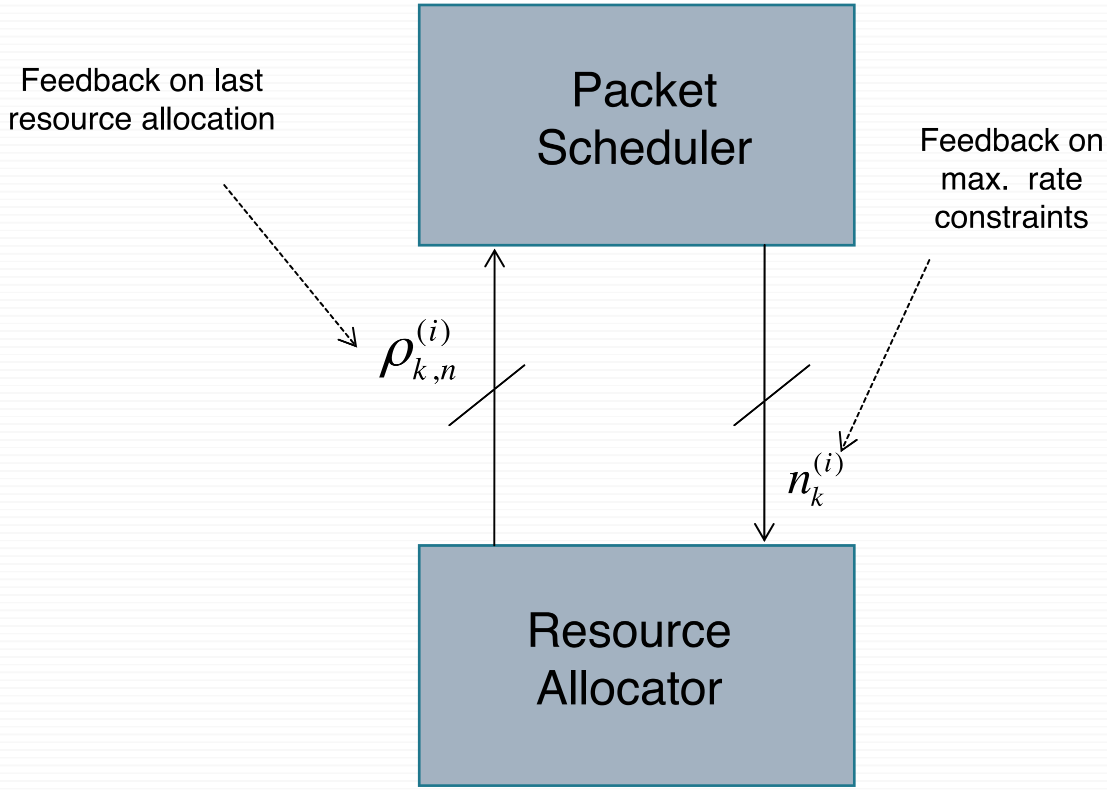
LP algorithm: packet scheduler

- By assigning a different number of subcarriers to users, the LP RRA sets the actual rate offered by the system to each user.
- Exploiting multi-user diversity, it tends to assign most of the resources to users with the best channels.
- In order to compensate the displacement of resources due to the RRA, in each cell we implement a Packet Scheduler (PS) that aims at maximizing fairness among users by setting the max number of resources for each user

LP algorithm: architecture

- At the beginning of each frame, in each cell the PS sets the maximum rate per user
- Given the requirements dictated by the PS, the RRA LP iterates until it finds a stable allocation in each cell
- If, after a certain number of iterations, a stable allocation has not been found, the load of the cells is progressively reduced until allocation converges.
- The allocation results are fed-back to the PS so that it updates the requirements to enforce fairness

LP algorithm: packet scheduler



Multi-cell algorithms: centralized schemes

Centralized approach

- The maximum achievable performance of multi-cell resource allocation is currently unknown.
- In analogy with the bound developed for the single-cell scenario, we develop a bound on the performance of centralized resource allocation in the dual domain.
  - Pros: Analytically sound, useful bound to compare other algorithms' performance
  - Cons: Exponential complexity, requires the knowledge of all the users channel gains at the central controller

Optimal bound: primal

- The primal is formulated as a global minimization problem
- The problem is combinatorial and its complexity grows exponentially with *K* and *N* and the number of cells
- To partially reduce the complexity we admit only a small set of possible transmission formats

$$\min_{\rho,p} \sum_{i} \sum_{n=1}^{N} \sum_{k=1}^{K} f_{k,n}^{(i)}\left(r_{k,n}^{(i)}\right) \rho_{k,n}^{(i)}$$

$$s.t.$$

$$\sum_{n=1}^{N} r_{k,n}^{(i)} \rho_{k,n}^{(i)} \geq r^{(i)}(k) \quad \forall i,k$$

$$R = \left\{ \rho_{k,n}^{(i)} \mid \sum_{k=1}^{K} \rho_{k,n}^{(i)} = 1 \quad \forall i,n; \rho_{k,n}^{(i)} \in \{0,1\} \right\}$$

Optimal bound: Lagrange dual

- The Lagrange dual of the RRA can be written as the sum of *N* reduced-complexity minimization problems

$$\sum_{n=1}^{N} \min_{r} \sum_{i} \sum_{k=1}^{K} \left(f_{k,n}^{(i)}\left(r_{k,n}^{(i)}\right) - \lambda_{k}^{(i)} r_{k,n}^{(i)} \right) + \sum_{i} \sum_{k=1}^{K} \lambda_{k}^{(i)} r^{(i)}(k)$$

$$s.t.$$

$$\sum_{k=1}^{K} \rho_{k,n}^{(i)} = 1 \quad \forall n,i$$

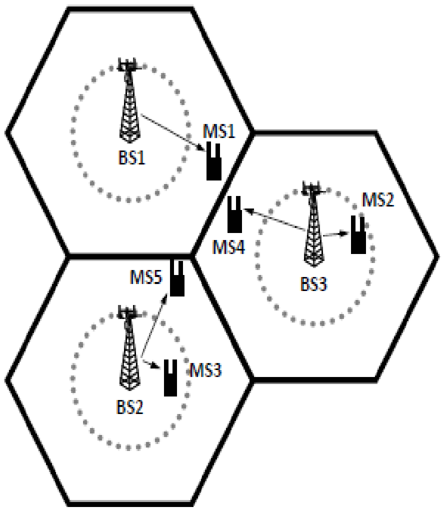
- Due to interference, the solution of the dual optimization problem needs an iterative strategy (even in the centralized approach!)

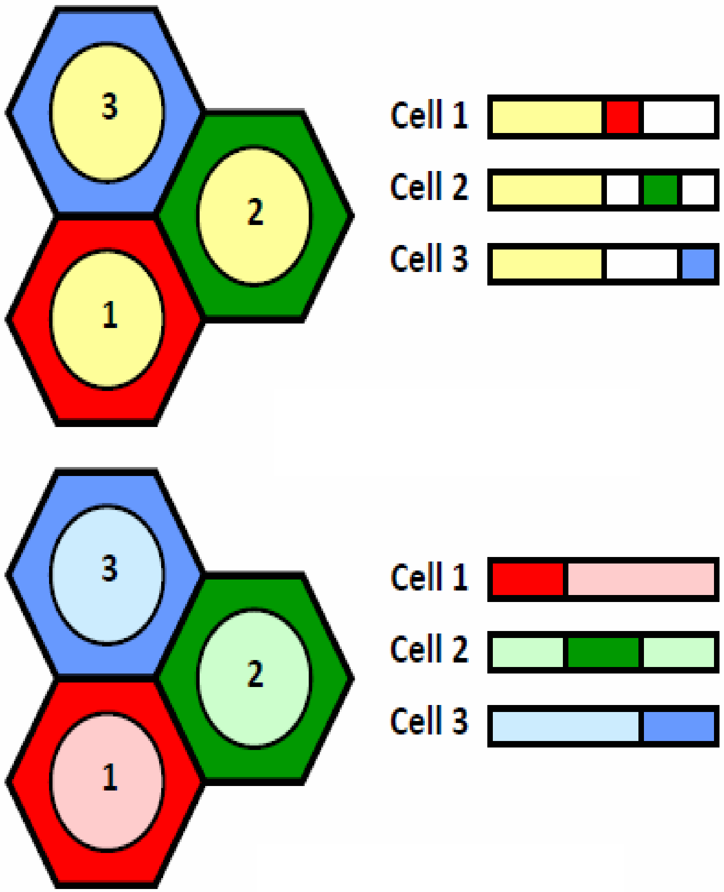
Optimal bound: Lagrange dual

- The main idea is to locally optimize via coordinate descent.
  - For each **λ**, we first find the optimal user and transmission format for cell #1 while keeping fixed the allocation in all other cells, then we find the optimal user and transmission format for cell #2 keeping all other fixed, and so on.
- Note that, during each iteration, only a small finite number of power levels need to be searched over.
- Such an iterative process is guaranteed to converge, because each iteration strictly decreases the objective function. The convergence point is guaranteed to be at least a *local minimum*.

FFR-A and FFR-B

- Static channel allocation according the pattern in figure





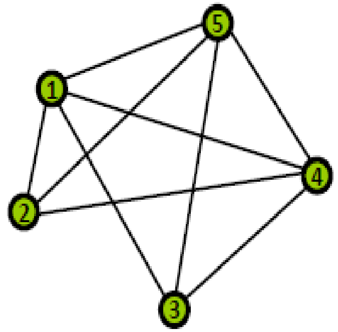
FFR-A and FFR-B

THE GRAPH CONSTRUCTION RULE FOR FFR-A

Node a and node b in the interference graph are connected by an edge if:

- A1. MS a and MS b are users of the same cell; or
- A2. MS a is a cell-edge user of cell i and MS b is a cell-edge user of cell j, where cell i and cell j are neighbors[a]; or
- A3. MS a is a cell-center user of cell i and MS b is a cell-edge user of cell j, or, MS a is a cell-edge user of cell i and MS b is a cell-center user of cell j, where cell i and cell j are neighbors.

Otherwise, node a and node b are not connected by an edge.

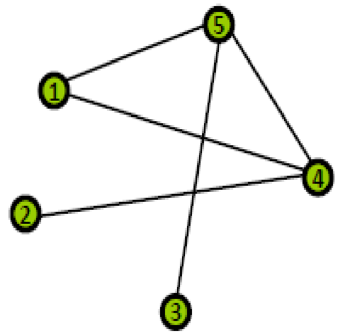


THE GRAPH CONSTRUCTION RULE FOR FFR-B

Node a and node b in the interference graph are connected by an edge if:

- B1. MS a and MS b are users of the same cell; or
- B2. MS a is a cell-edge user of cell i and MS b is a cell-edge user of cell j, where cell i and cell j are neighbors.

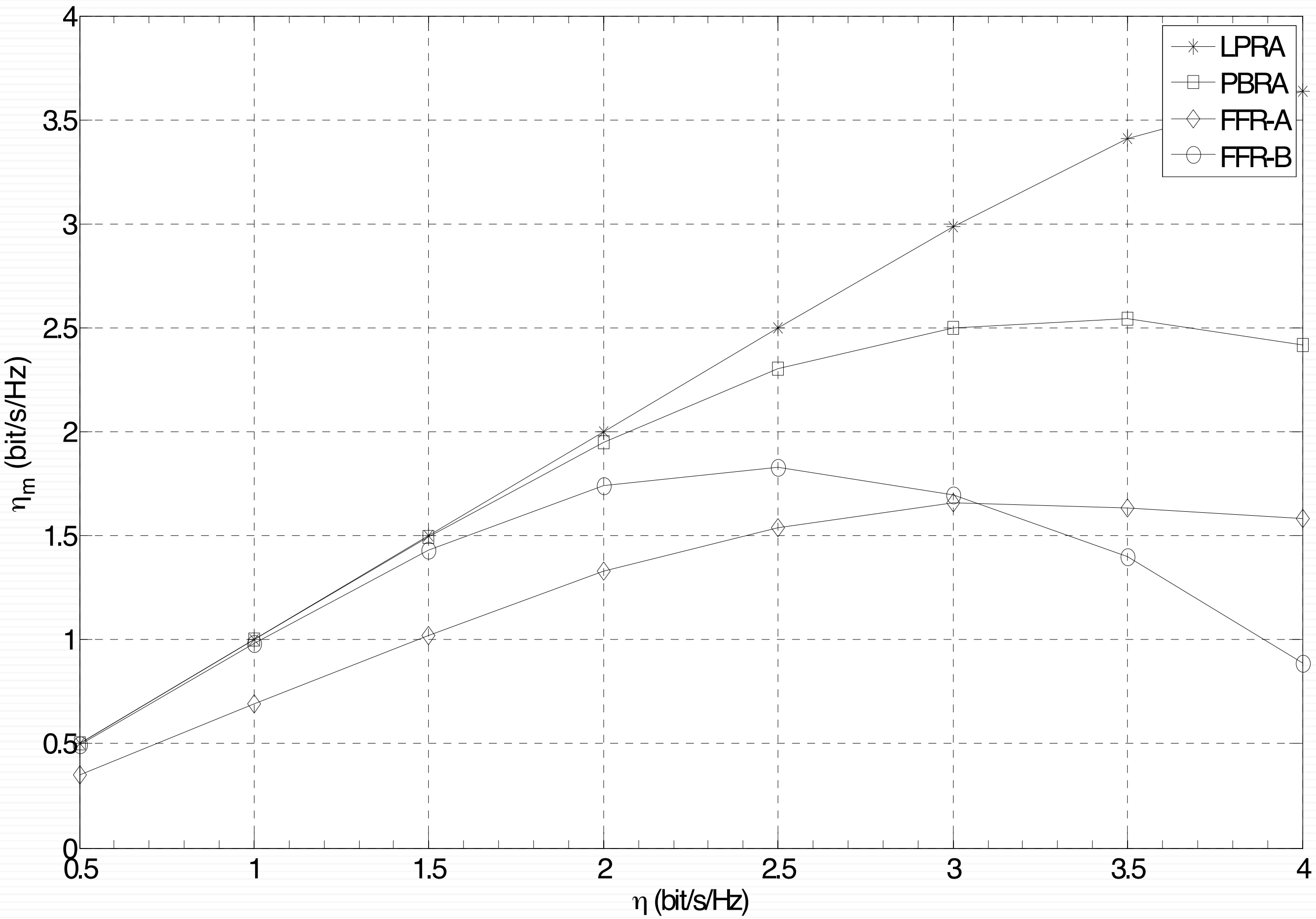
Otherwise, node a and node b are not connected by an edge.



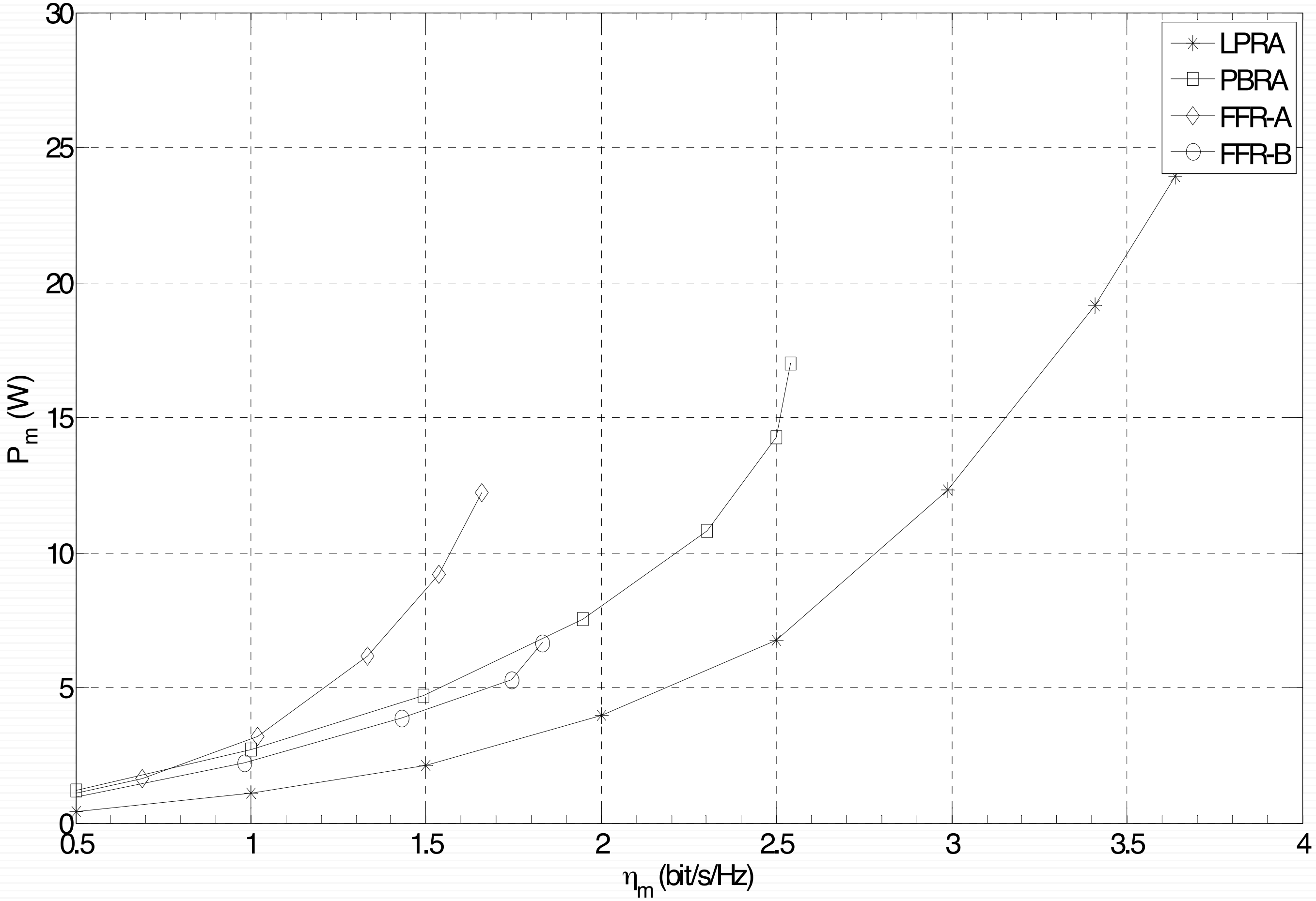
Simulation setup

- Number of cells = 7
- Radius of each cell R = 500 m
- Total available bandwidth W = 5MHz
- Center frequency = 2 GHz
- Number of subcarriers N = 16
- Number of users K = 8

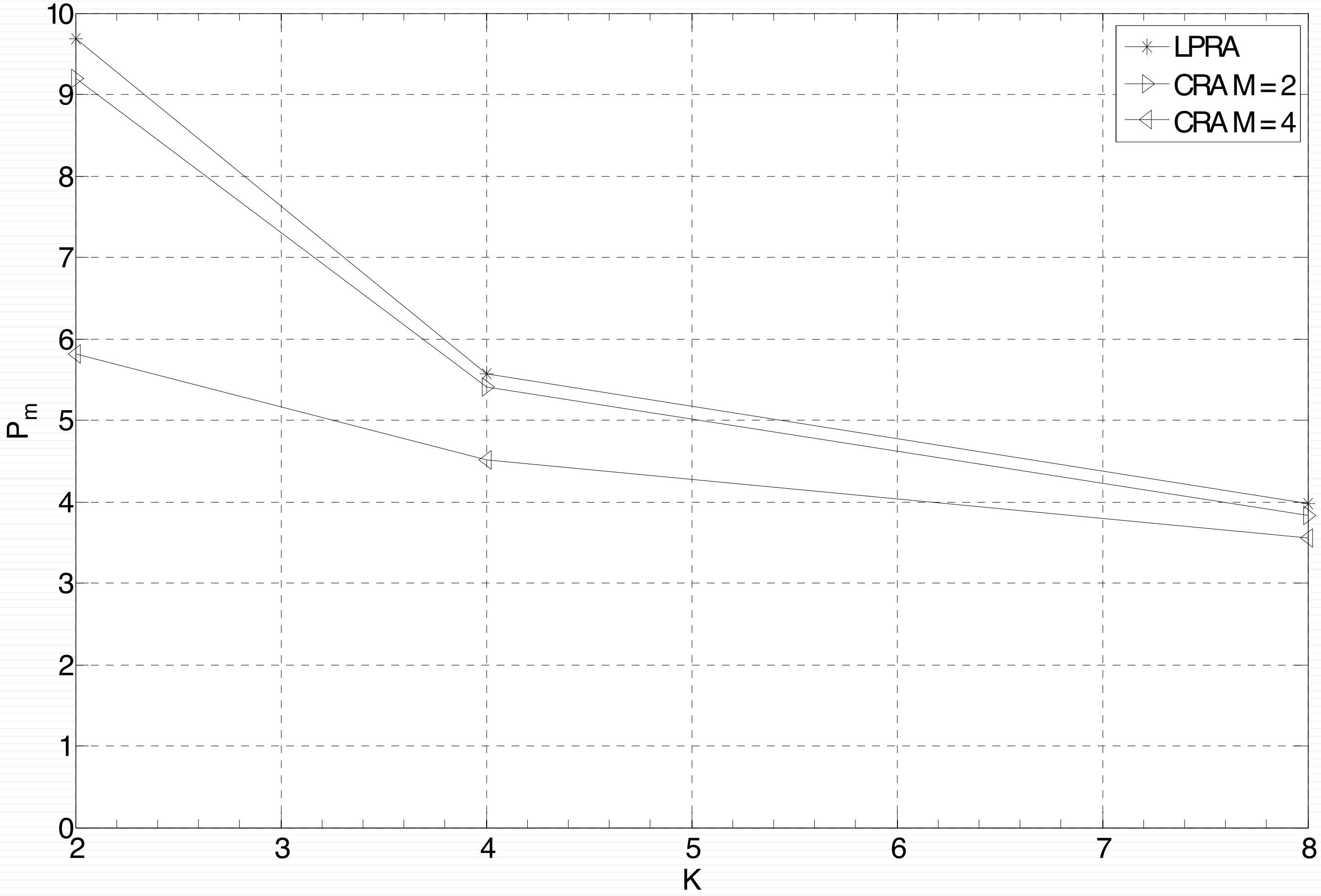
Measured spectral efficiency



Power vs. spectral efficiency



Power vs. Number of users



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